

# Effects of Dipole-Dipole Interaction on the Transmitted spectrum of Two-level Atoms trapped in an optical cavity

Yuqing Zhang<sup>1</sup>, Lei Tan<sup>1,2,‡</sup>, Peter Barker<sup>2</sup>

<sup>1</sup>Institute of Theoretical Physics, Lanzhou University, Lanzhou 730000, China

<sup>2</sup>Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, United Kingdom

**Abstract.** The transmission spectrum of two dipole-dipole coupled atoms interacting with a single-mode optical cavity in strong coupling regime is investigated theoretically for the lower and higher excitation cases, respectively. The dressed states containing the dipole-dipole interaction (DDI) are obtained by transforming the two-atom system into an effective single-atom one. We found that the DDI can enhance the effects resulting from the positive atom-cavity detunings but weaken them for the negative detunings cases for lower excitation, which can promote the spectrum exhibiting two asymmetric peaks and shift the heights and the positions of them. For the higher excitation cases, DDI can augment the atomic saturation and lead to the deforming of the spectrum. Furthermore, the large DDI can make the atom and the cavity decouple, making a singlet of the normal-mode spectrum.

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## 1. Introduction

Realization of the strong coupling regime (SCR) in cavity quantum electrodynamics has opened a new vision for the exploration of quantum mechanics [1, 2]. SCR is very attractive, as photons emitted by atoms inside the cavity mode can be reabsorbed and reemitted, etc., leading to Rabi oscillations [3], which gives rise to a normal-mode splitting in the eigenvalue spectrum of the atom-cavity system [4, 5, 6]. The normal-mode splitting has been observed with atomic beams passing a cavity in both the microwave regime [7, 8, 9] and optical regime [10], as well as with trapped atoms in optical cavities [11, 12, 13, 14]. In experiment, the normal-mode splitting is detected by probing the transmitted spectrum in low excitation. With increased excitation, the spectrum presents hysteresis, and then forms a close structure. When the atoms are saturated, they decouple from the cavity and only a single peak appears in the spectrum [15, 16].

‡ E-mail : lei.tan@ucl.ac.uk

However, few previous studies have considered the DDI, even though many works have been explored atom-cavity systems with DDI [17, 18, 19, 20, 21, 22, 23]. In fact, the DDI can profoundly affect the light absorption and lead to the shift of the atomic energy levels [24]. The renormalization of the atomic resonance frequency due to the DDI can result in the optical bistability of the atomic system [25, 26]. So it is natural to expect that the transmitted spectrum may present some novel characteristics due to the DDI. Fortunately, in recent years substantial progress towards the study about solid materials [27, 28] and ultracold atoms [29, 30, 31, 32] proves a good platform for the study of DDI. In this work, we go one step further and investigate the transmitted spectrum of two dipole-dipole coupled atoms trapped and strongly coupled to an optical resonator. The behavior of spectrum in steady-state is studied for a wide range of DDI intensity and atom-cavity detuning. The relation and distinction of their effects on the spectrum both in weak excitation limit and higher excitation are also explored.

The paper is organized as follows: Sec.2 presents the theoretical model under consideration and provides the steady-state solution by solving the master equation. Sec.3 is devoted to the study of the transmitted spectrum in the weak excitation limit. Sec.4 describes the structure characteristics of transmitted spectrum for a strong driving intensity. The effects of both the detuning and the DDI on the spectrum are discussed. Finally, we present our conclusions in Sec.5.

## 2. Model

We consider two identical dipole-dipole coupled two-level atoms interacting with a single-mode high-finesse optical cavity. The system is pumped along the cavity axis by a coherent laser field of frequency  $\omega_p$  and an effective amplitude  $\eta$ . The Hamiltonian for the system in the rotating wave and electric dipole approximations is given by [23]

$$H = -\Delta_c a^\dagger a - \sum_{k=1}^2 [\Delta_a \sigma_k^\dagger \sigma_k - g(a^\dagger \sigma_k + a \sigma_k^\dagger)] + J(\sigma_1^\dagger \sigma_2^\dagger + \sigma_2^\dagger \sigma_1) + \eta(a + a^\dagger) \quad (1)$$

where  $\Delta_c = \omega_p - \omega_c$ ,  $\Delta_a = \omega_p - \omega_a$ .  $\omega_c$  and  $\omega_a$  are the resonance frequencies of the atoms and the cavity field, respectively.  $a^\dagger$  and  $a$  are the field creation and annihilation operators,  $\sigma_k^\dagger$  and  $\sigma_k$  represent the raising and lowering operators of the atom  $k$  ( $k = 1, 2$ ). The first term of Hamiltonian (1) is the free Hamiltonian of cavity. The atomic free Hamiltonian and the interaction Hamiltonian of atoms and cavity with coupling strength  $g$  are shown in the second term. The third term describes the DDI between atoms and the last term is the pump field Hamiltonian. The DDI is defined in the form [18]

$$J = \frac{3}{4}(\Gamma_0 c^3 / \omega_a^3 r^3)(1 - 3 \cos^2 \varphi) \quad (2)$$

where  $r$  is the distance between the atoms and  $\varphi$  is the atomic dipole moments with respect to the interatomic axis.  $\Gamma_0$  denotes the atomic spontaneous emission rate in

free space. Here, we assume the dipole moments of the two atoms are parallel to each other and are polarized in the direction perpendicular to the interatomic axis. Then,  $\cos \varphi = 0$ , the DDI intensity only depends on the positions of the two atoms in the cavity.

Dissipations results from excitations spontaneous emission and cavity photonic leakage can be taken into account within the quantum master equation of density matrix  $\rho$ . It is expressed in the usual Lindblad form in Born-Markov approximation ( $\hbar = 1$ ) [33]

$$\begin{aligned}\dot{\rho} &= -i[H, \rho] + L_{\kappa}\rho + L_{\gamma}\rho + L_{\gamma'}\rho \\ L_{\kappa}\rho &= \kappa[2a\rho a^{\dagger} - a^{\dagger}a\rho - \rho a^{\dagger}a] \\ L_{\gamma}\rho &= \sum_{k=1}^2 \gamma(2\sigma_k\rho\sigma_k^{\dagger} - \sigma_k^{\dagger}\sigma_k\rho - \rho\sigma_k^{\dagger}\sigma_k) \\ L_{\gamma'}\rho &= \gamma'(2\sigma_1\rho\sigma_2^{\dagger} - \sigma_1^{\dagger}\sigma_2\rho - \rho\sigma_1^{\dagger}\sigma_2) \\ &\quad + \gamma'(2\sigma_2\rho\sigma_1^{\dagger} - \sigma_2^{\dagger}\sigma_1\rho - \rho\sigma_2^{\dagger}\sigma_1)\end{aligned}\quad (3)$$

Here, the non-unitary parts  $L_{\kappa}\rho$ ,  $L_{\gamma}\rho$  and  $L_{\gamma'}\rho$  describe the coupling of the field mode and the atoms to the environment. The coefficients  $\kappa$  and  $\gamma$  are the decay rates of the cavity field and the atoms respectively. The atom-atom cooperation induced by their coupling with a common reservoir is given by  $\gamma'$  [18, 21, 34], which is important only when the atomic distances are small relative to the radiation wavelength.

The time evolution of the operators expectation values for the atom-cavity system can be obtained with the master equation

$$\langle \dot{a} \rangle = i(\tilde{\Delta}_c \langle a \rangle - g \langle \sigma_1 \rangle - g \langle \sigma_2 \rangle - \eta) \quad (4)$$

$$\langle \dot{\sigma}_1 \rangle = i(\tilde{\Delta}_a \langle \sigma_1 \rangle + g \langle a \sigma_{1z} \rangle + \tilde{J} \langle \sigma_1 \rangle \langle \sigma_{2z} \rangle) \quad (5)$$

$$\langle \dot{\sigma}_2 \rangle = i(\tilde{\Delta}_a \langle \sigma_2 \rangle + g \langle a \sigma_{2z} \rangle + \tilde{J} \langle \sigma_2 \rangle \langle \sigma_{1z} \rangle) \quad (6)$$

$$\langle \dot{\sigma}_{1z} \rangle = 2ig(\langle a^{\dagger} \sigma_1 \rangle - \langle a \sigma_1^{\dagger} \rangle) - 2\gamma(1 + \langle \sigma_{1z} \rangle) \quad (7)$$

$$\langle \dot{\sigma}_{2z} \rangle = 2ig(\langle a^{\dagger} \sigma_2 \rangle - \langle a \sigma_2^{\dagger} \rangle) - 2\gamma(1 + \langle \sigma_{2z} \rangle) \quad (8)$$

where  $\tilde{\Delta}_a = \Delta_a + i\gamma$ ,  $\tilde{\Delta}_c = \Delta_c + i\kappa$ , and  $\tilde{J} = J - i\gamma'$ .

Then we can get the steady state solutions of Eqs.(4)-(6) by setting  $\langle \dot{a} \rangle = \langle \dot{\sigma}_1 \rangle = \langle \dot{\sigma}_2 \rangle = \langle \dot{\sigma}_{1z} \rangle = \langle \dot{\sigma}_{2z} \rangle = 0$ .

When  $g \gg (\gamma, \kappa)$ , the atom-cavity system reaches a strong coupling regime. The new eigenstates of the system are described by the dressed states, which are linear combination of pairs of bare atom states and cavity field state. However, it is difficult to find out the dressed states of the atom-cavity system with two dipole-dipole coupled atoms. In fact, when the excitation of the atoms is very low, we can adopt the methods in [18] and simplify the two-atom system to an effective single atom system. Then the effective form of Hamiltonian  $H$  in Eq.(1) can be written as

$$\begin{aligned}H_{eff} &= -\Delta_c a_1^{\dagger} a - (\Delta_a - J) \sigma_1^{\dagger} \sigma_1 \\ &\quad + \sqrt{2}g(a^{\dagger} \sigma_1 + a \sigma_1^{\dagger}) + \eta(a + a^{\dagger}).\end{aligned}\quad (9)$$

In the transformed Hamiltonian, the dipole coupled atoms are denoted by two fictitious atoms. Only one of them couples to the field mode with frequencies  $\omega_a + J$  and an effective coupling strength  $\sqrt{2}g$ , but the other atom freely evolves decoupling from the field. As a result, the dressed states of the transformed system are similar to that of the single-atom system

$$\begin{aligned} |0\rangle &= |g\rangle|0\rangle, \\ |n_-\rangle &= \sin \frac{\theta_n}{2} |e, n-1\rangle - \cos \frac{\theta_n}{2} |g, n\rangle, \\ |n_+\rangle &= \cos \frac{\theta_n}{2} |e, n-1\rangle + \sin \frac{\theta_n}{2} |g, n\rangle. \end{aligned} \quad (10)$$

where  $\sqrt{n}$  is a photon number state,  $\theta_n = \arctan 2\sqrt{2}g\sqrt{n}/(\Delta + J)$ ,  $\Delta = \omega_a - \omega_c$  is the detuning between atom and field. The corresponding eigenenergies are

$$\begin{aligned} E_0 &= 0, \\ E_{n\pm} &= \omega_c + \frac{\Delta + J}{2} \pm \frac{1}{2} \sqrt{(\Delta + J)^2 + 8g^2n}. \end{aligned} \quad (11)$$

Spectrum of the first doublet of these states in a degenerate system (for  $\omega_a = \omega_c$ ) splits into two new resonances, called normal-mode or vacuum-Rabi splitting. Observation of the normal-mode splitting, in fact, is also a benchmark signature that a system has reached the SCR of cavity QED.

To investigate the steady state normal-mode spectrum, we introduce the intracavity photon number [35],

$$\langle a^\dagger a \rangle_0 = |\langle a \rangle_0|^2 \quad (12)$$

which is given by the modulus square of  $\langle a \rangle_0$  and is sufficient to calculate a spectrum of the coupled atoms-cavity system.

### 3. Normal-mode spectrum in low excitation limit

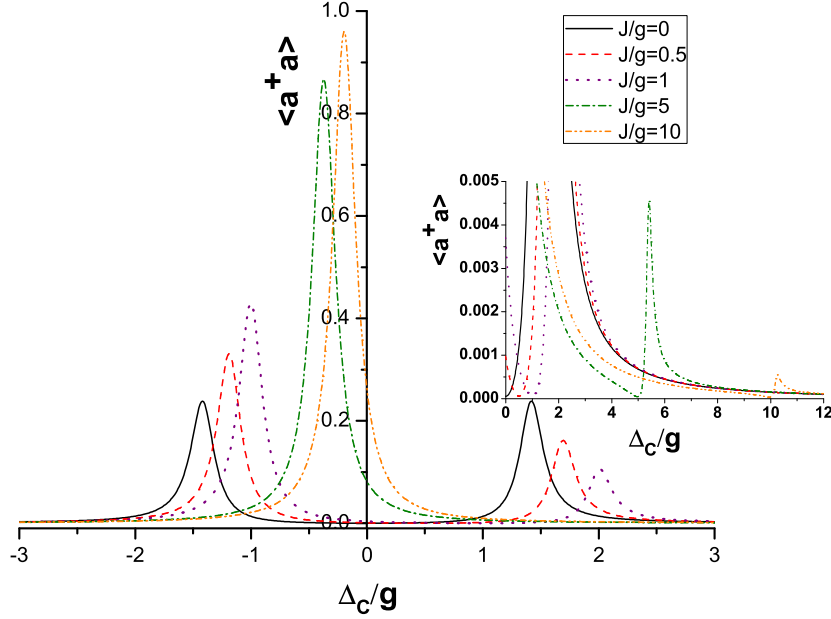
By choosing an appropriate pump beam, we can keep a weak pump intensity and thus a low atomic excitation can be achieved. In this condition,  $\langle \sigma_{1z} \rangle, \langle \sigma_{2z} \rangle \rightarrow -1$ , so we can set  $\langle a\sigma_{1z} \rangle = \langle a\sigma_{2z} \rangle = -\langle a \rangle$ . Then the steady-state solution of Eqs.(4),(5) and (6) and the steady state intracavity photon number can be given:

$$\langle a \rangle_0 = \frac{\eta}{\tilde{\Delta}_c} \cdot \frac{1}{1-v} \quad (13)$$

$$\langle \sigma_1 \rangle_0 = \frac{\eta v}{g} \cdot \frac{1}{1-v} \quad (14)$$

$$\langle \sigma_2 \rangle_0 = \frac{\eta v}{g} \cdot \frac{1}{1-v} \quad (15)$$

$$\langle a^\dagger a \rangle_0 = \frac{\eta^2}{|\tilde{\Delta}_c|^2} \cdot \frac{1}{|1-v|^2}, \quad (16)$$



**Figure 1.** The normal-mode spectra for different DDI intensities is shown. The parameters are  $\Delta = 0$ ,  $(\eta, \kappa, \gamma, \gamma') = (0.12, 0.12, 0.0767, 0.05)g$ .

where

$$v = \frac{2g^2}{\tilde{\Delta}_c[\tilde{\Delta}_a - \tilde{J}]} \quad (17)$$

These results are based on the classical approximation [35], which converts the fermionic commutation relation to a bosonic form, treating both atoms and field as linear harmonic oscillators and omitting the effects of saturation.

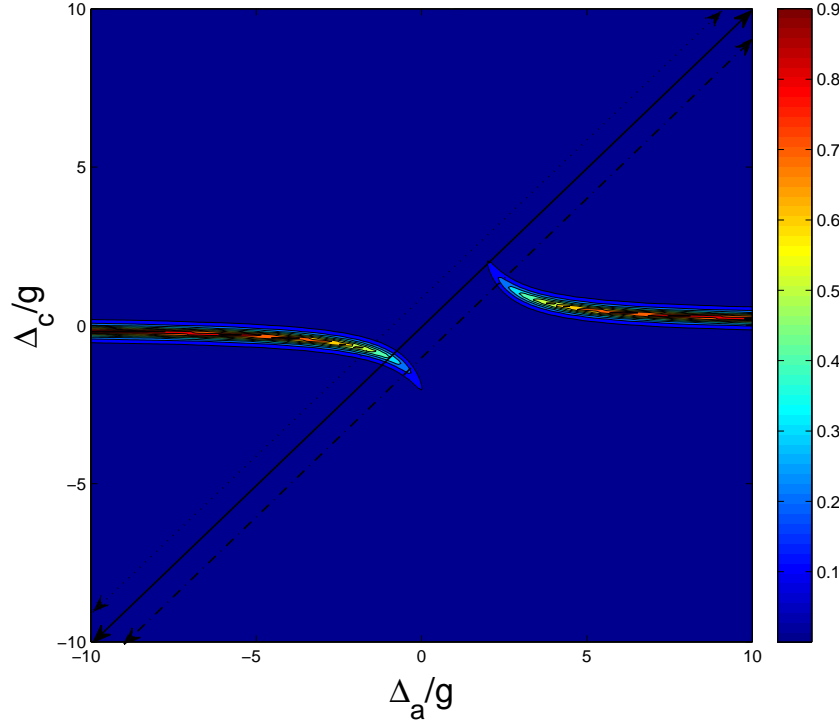
The two normal-mode resonances are characterized by the eigen-frequencies  $\omega_{\pm}$ ,

$$\omega_{\pm} = -\frac{1}{2}(\tilde{\Delta}_a - \tilde{J} + \tilde{\Delta}_c) \pm \frac{1}{2}\sqrt{[8g^2 + (\tilde{\Delta}_a - \tilde{J} - \tilde{\Delta}_c)^2]}, \quad (18)$$

which are based on the Eq.(9). The frequencies  $\omega_{\pm}$  have complex values. The real part  $\text{Re}(\omega_{\pm})$  determines the position of the resonances, while the imaginary part describes their widths. However, in the strong-coupling regime,  $g \gg (\gamma, \gamma', \kappa)$ , so the effects of the decay on the position of the resonances can be neglected. The resonance frequencies  $\omega_{\pm}$ , in this condition, match the first pair of dressed states in Eq.(11). For  $\Delta = J = 0$ , the distance between the two resonances has a minimum value  $\omega_+ - \omega_- \approx 2\sqrt{2}g$ . When  $\Delta$  and  $J$  are nonzero, the position and distance of them can be obtained from the expression of Eq.(11).

As the two-atom system to some extent is analogous to the one atom system, we cite the parameters value in [36], in which photon blockade for the light transmitted by an optical cavity containing one trapped atom is observed. The decay rate  $\gamma'$  due to the DDI is usually weak, so without loss of generality, we take it as 0.05g in this section.

In Fig.1, the normal-mode spectra for different DDI intensities is plotted. From Fig.1 we can find that when  $J = 0$ , the amplitudes of both resonances are equal and



**Figure 2.** The normal modes form an avoided crossing between the resonances of the bare atoms and the bare cavity. Three different cases of  $\Delta/g = 1$  (dashed-dotted),  $\Delta = 0$  (solid) and  $\Delta/g = -1$  (dotted) are shown. The dipole-dipole interaction strength is taken as  $J/g = 1$ . The parameters are  $(\eta, \kappa, \gamma, \gamma') = (0.12, 0.12, 0.0767, 0.05)g$ .

the position of them is symmetric about  $\Delta_c = 0$  with a minimum distance about  $2\sqrt{2}g$ . However, when  $J \neq 0$ , it indicates that with the increase of DDI intensity, the left peak becomes higher and gets closer to  $\Delta_c = 0$ , while the height of right peak is reduced greatly and it gets far away from  $\Delta_c = 0$ . Moreover, the distance between the two peaks shows an enlarged tendency. This is because for  $\Delta = 0$ ,  $J \neq 0$  the distance between the two peaks is in accord with the expression  $\sqrt{J^2 + 8g^2}$ , which is a monotone increasing function of  $J$ . In addition, the height of them are determined by the value of  $\sin \frac{\theta_n}{2}$  and  $\cos \frac{\theta_n}{2}$  in Eq.(10). When  $\Delta = J = 0$ , the contributions from the atoms and the cavity states are equal so that the normal modes have the same height. With the increase of DDI intensity, the excitation probability of the bare cavity field state enhances for the lower dressed state  $|1_-\rangle$ , while for the bare atoms states it reduces greatly. The results are opposite for the higher state  $|1_+\rangle$ . It should be noticed that the system is pumped by a coherent laser beam shining on one of the cavity mirrors, so the bare cavity states are more easily excited, leading to a better visibility of the “cavity-like” peak. However, when  $J \gg g$ , the probabilities  $\sin^2(\frac{\theta_n}{2}) \approx 0$ ,  $\cos^2(\frac{\theta_n}{2}) \approx 1$ , so the system almost is in the state  $|g, 1\rangle$ . The atoms are not being excited in this case and the spectrum shows a single peak.

In Fig.2, the normal modes spectrum are revealed as functions of  $\Delta_a$  and  $\Delta_c$ . The atom-cavity detuning is defined as  $\Delta = \omega_a - \omega_c$ . An avoided crossing between the

atomic transition and the resonant frequency of the cavity is shown. With the increases of  $\Delta_a$  and  $\Delta_c$ , the atom-cavity system decouples and the two resonances approach the eigenfrequencies of the atoms and the cavity asymptotically .

In Fig.3(a), when  $\Delta \neq 0$  both the position and height of the two resonances shift. On the one hand, according to Eq.(11), when  $J = 0$ ,  $\Delta \neq 0$ , is it obvious that the position of the two peaks rely on the value of  $\Delta$  and the distance between them depends on  $\sqrt{\Delta^2 + 8g^2}$ . On the other hand, the excitation of a dressed state is determined by the contribution of the cavity state to the dressed state. For positive detuning, based on Eq.(10), the excitation probability of the bare cavity field state of  $|1_-\rangle$  augments with the increase of  $\Delta$ , but for  $|1_+\rangle$  it reduces. However, the results are opposite for negative detuning. Therefore, when  $\Delta \neq 0$  the two resonances are better to exhibit “cavity-like” form with enlarged separation.

Interestingly, as is reflected in Figs.1 and 3(a), it indicates that the DDI plays a similar role as the positive detuning. The effect of the DDI is equivalent to increasing the positive detunings and decreasing the negative detunings. To confirm this conclusion, in Fig.3(b) the cooperative action of  $\Delta$  and  $J$  on the two resonances is revealed. It shows that the two resonances are symmetry with equal height when  $\Delta + J = 0$ . For  $\Delta > 0$ ,  $\Delta$  and  $J$  have consistent effects on the two peaks. While for  $\Delta < 0$ , the influences of them cancel each other, and the practical states rely on the larger absolute value one of them. In fact, these results are apparent. Both for the dressed states in Eq.(10) and the corresponding eigenenergies in Eq.(11),  $\Delta$  and  $J$  are present with the form “ $\Delta + J$ ”. Therefore, the position and height of the two peaks, as well as the distance between them all depend on the cooperative action of  $\Delta + J$ .

#### 4. Transmission spectrum for higher pump intensity

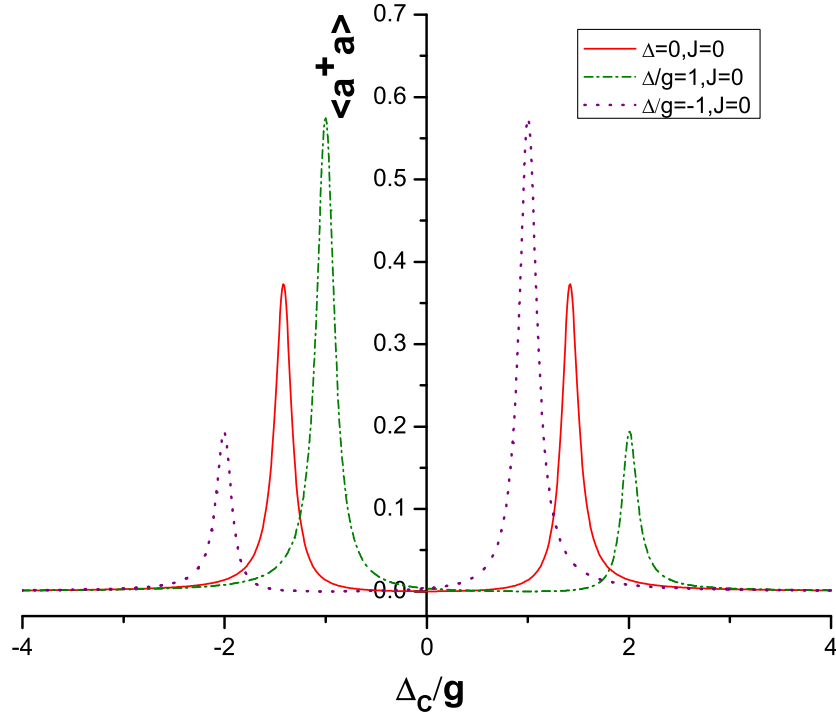
We have remarked that the validity of above results depends on the assumption of the weak excitation. For higher pump intensity, the atomic saturation can not be neglected. We can define  $\langle \sigma_z \rangle_0 = -\frac{1}{1+s_0}$ , and treat the cavity field classically by replacing  $a$  with  $\langle a \rangle$ . Then  $\langle a \sigma_z \rangle$  can be written as a product form  $\langle a \rangle \langle \sigma_z \rangle$  and the steady state of Eqs.(3),(4) and (5) can be calculated.

$$\langle a \rangle_0 = \frac{\eta}{\tilde{\Delta}_c} \cdot \frac{1}{1 - \mu} \quad (19)$$

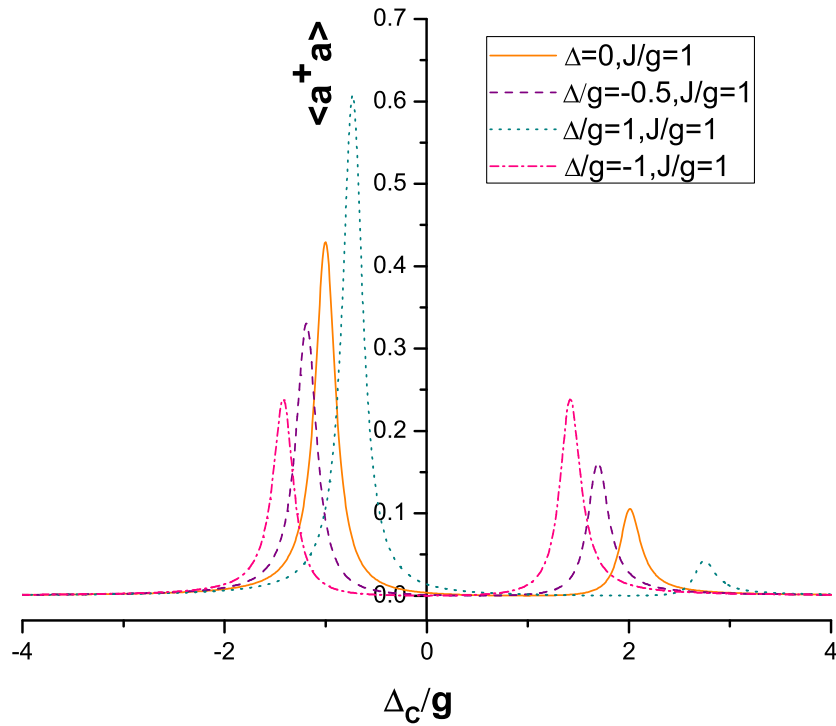
$$\langle \sigma \rangle_0 = \frac{\eta v}{g} \cdot \frac{1}{1 - \mu} \quad (20)$$

$$\mu = \frac{2g^2}{\tilde{\Delta}_c[\tilde{\Delta}_a(1 + s_0) - \tilde{J}]} \quad (21)$$

$$s_0 = \frac{2g^2(1 + s_0)^2 \langle a^\dagger a \rangle_0}{|\tilde{\Delta}_a(1 + s_0) - \tilde{J}|^2} + \frac{2g^2(1 + s_0) \langle a^\dagger a \rangle_0 \gamma'}{|\tilde{\Delta}_a(1 + s_0) - \tilde{J}|^2 \gamma} \quad (22)$$



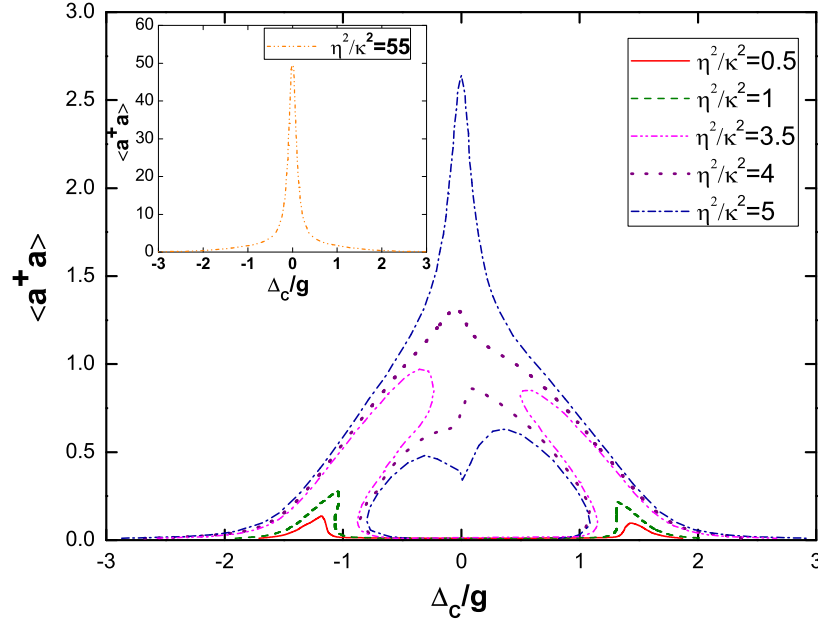
(a)



(b)

**Figure 3.** The normal-mode spectrums for different values of  $\Delta$  and  $J$  are plotted. (a) There is no dipole-dipole interaction between atoms, that is  $J=0$ ,  $\gamma'=0$ . (b) Both the detuning and the dipole-dipole interaction are considered. The dipole-dipole interaction intensity is taken as  $J/g = 1$ . Other parameters are  $(\eta, \kappa, \gamma, \gamma') = (0.12, 0.12, 0.0767, 0.05)g$ .

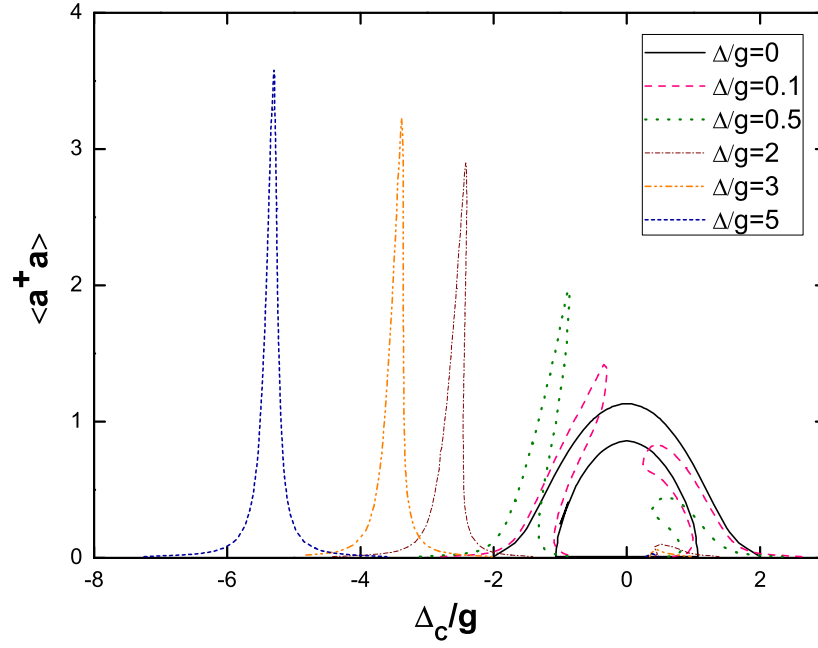




**Figure 4.** The normal mode structure is revealed for different pump intensities. With increase of pump intensities, atoms tend to saturation, and the two peaks bend towards the center. In the limit of the high excitation, the atoms are saturated and the spectrum shows a single peak, as is shown in the inset.  $J=0.5g$ ,  $\kappa=\gamma=0.1g$ ,  $\Delta = 0$ ,  $\gamma'=0.01g$ .

where  $s_0$  is the saturation parameter. Notice that  $s_0 \rightarrow 0$  corresponds to the low saturation limit.

In Fig.4, with Eqs.(19),(21) and (22) the spectrum for DDI atoms with increased pump intensity are plotted. It shows similar behavior as the system without DDI [15]. These results are not surprising, because the dipole-coupled two atoms can be treated as an effective atom with renormalized atomic frequency and atom-cavity coupling intensity. With the increase of the pump intensity, atoms begin to saturate and the peaks of the two resonances shift their position and deform, bending towards the center. Finally, they meet and form a closed structure. It is important to note that there are three possible expectation values for the operator  $\langle a^\dagger a \rangle_0$  when  $\eta^2/\kappa^2 \geq 1$ . One of them is unstable but the other two are stable. The amplitude of intracavity field can switch between the two stable values, named bistability, which predicts a nonlinear relation between input and output intensity. The system, in this case, evolves from two coupled harmonic oscillators to highly deformed anharmonic oscillators. In the limit of high excitation, as is shown in the inset, the atoms are saturated and do not contribute significantly to the dynamics of the system. The spectrum resembles that of an empty cavity, evolving from two peaks to a singlet. However, the spectrum no longer shows a symmetrical shapes as system without DDI. Because we introduce the DDI into system, the shape of these curves are asymmetrical about  $\Delta_c = 0$ .



**Figure 5.** The effects of the atom-cavity detuning on the normal mode structure of high excitation. The spectrum deformed, and changes to a singlet in the limit of large detuning.  $\eta^2/\kappa^2 = 4$ ,  $J = 0$ ,  $\kappa=\gamma=0.1g$ ,  $\gamma'=0.1g$ .

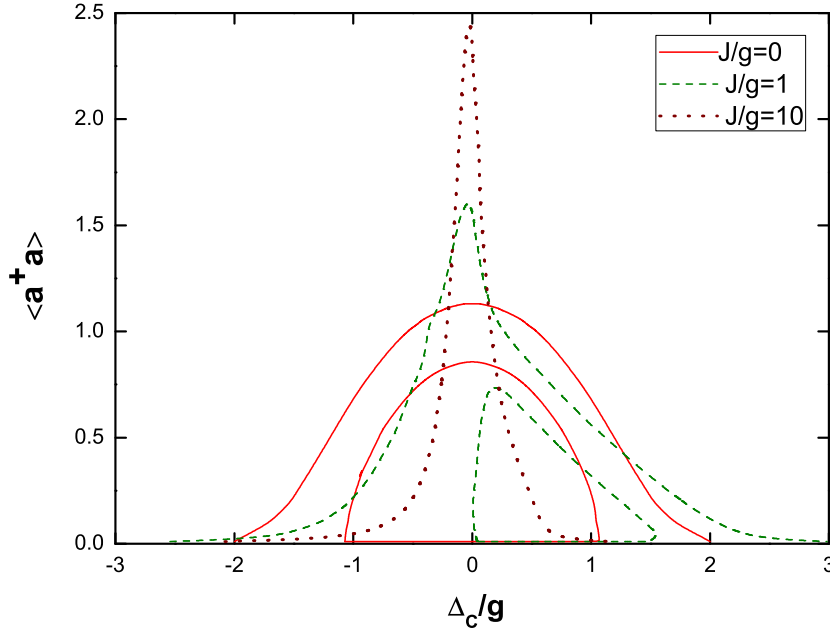
In weak excitation limit, the DDI influences both the height and position of the two peaks and has similar effects as the positive detunings. So for higher excitation, the effects of  $\Delta$  and  $J$  on the spectrum are also worth exploring. In the following section we begin from the spectrum with closed structure in higher excitation to study their effects.

In Fig.5, a interesting phenomenon arises with the increase of atom-cavity detuning. The original closed spectrum begins to separate and splits into two peaks. This can be inferred from Eqs.(19) and (22). Based on these two equations, we can get

$$s_0 = \frac{2g^2\eta^2(1+s_0)^2(1+\frac{\gamma'}{\gamma})}{|\tilde{\Delta}_c[(\Delta_c - \Delta + i\gamma)(1+s_0) - \tilde{J}] - 2g^2|^2}. \quad (23)$$

When  $J = 0$ , with increase of  $\Delta$ , the atomic saturation parameter  $s_0$  reduces. It is not difficult to understand that the detuning can make the atom-cavity coupling becomes weaken, thus reduces the atomic saturation. So with the increase of  $\Delta$ , the spectrum is gradually returning to the cases of weaker excitation. When  $\Delta/g \simeq 5$  the right peak nearly vanishes, while the right peak is more distinct. The system, in this case, is mainly dominated by  $\Delta$  and shows a decoupled tendency.

However, in Fig.6 the spectrum as a function of  $J$  for higher excitation presents a different behavior. The original closed spectrum only deforms but does not separates. This result maybe explained as follows. As mentioned above, the DDI can make the atomic frequency renormalized and change the atom-cavity coupling intensity from  $g$  to  $\sqrt{2}g$ . According to Eq.(23),  $s_0$  increases with the increase of  $J$  for  $\Delta = 0$ . For



**Figure 6.** The effects of dipole-dipole interaction on the normal mode structure of high excitation. The spectrum deformed, and changes to a singlet in the limit of high dipole-dipole interaction intensity.  $\eta^2/\kappa^2 = 4$ ,  $\Delta = 0$ ,  $\kappa=\gamma=0.1g$ ,  $\gamma' = 0.1g$ .

high excitation, the renormalization of the atomic frequency resulting from the small DDI does not generate pronounced influences on the excitation of the system, then the spectrum only deforms. However, for  $J \gg g$  cases, a large atom-cavity detuning forms due to the renormalization of the atomic frequency and plays a dominant role in the atom-cavity system. Then photons coming from a cavity mirror can not populate on the atomic states, making the spectrum shows a singlet.

## 5. Conclusion

We have characterized the transmission spectrum properties of two dipole-coupled two-level atoms strongly coupling to a single-mode optical cavity. In the low excitation limit, the DDI, acting as the atom-cavity detuning, can change the position and height of the two peaks. However, the DDI has similar effects as the positive detuning, and shows opposite effects as the negative detuning. The dressed states have also been derived by transforming the two-atom system to an effective single-atom system. For higher excitation, the atom-cavity detuning can reduce the atomic saturation, making the original closed structure separate. While the DDI can augment the atomic saturation, leading to the deforming of the original closed structure. Interestingly, except for the limit of high excitation and large detuning, the strong DDI can also result in the decoupling of the atoms and cavity, which leads to the spectrum showing a singlet. We expect that these results will be useful in understanding the quantum electrodynamics of the atom-cavity system with DDI.

## 6. Acknowledgement

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